Whole Numbers:

Unit 1 Whole numbers

Properties of whole numbers

Whole numbers are all the positive numbers, including 0, that are not fractions or decimals.

Whole numbers can be referred to by the symbol N_0. We can describe whole numbers by using set notation as $N_0 = \{0; 1; 2; 3; \ldots\}$

> N is the symbol for the natural numbers. Whole numbers are all the natural numbers including 0, which is why the symbol is N_0.

The commutative property

The commutative property refers to switching numbers when adding and multiplying. Changing the order of the numbers in addition or multiplication expressions will not change the answer, for example:
$4 + 2 = 6$ is the same as $2 + 4 = 6$
$3 \times 7 = 21$ is the same as $7 \times 3 = 21$

Exercise 1.1

Consider the six expressions below:

1 + 3	5 + 2	8 + 2	2 × 4	8 × 3	5 × 4

For each question below write true if it is equivalent and false if it is not equivalent to one of the above expressions.

1. $2 + 3$
2. 8×2
3. 4×5
4. $5 + 4$
5. 3×8
6. $2 + 5$

The associative property

The associative property refers to grouping numbers, and again this only applies to addition and multiplication. How the numbers are grouped together in addition or multiplication expressions will not change the answer, for example:

$(5 + 2) + 7 = 14$ is the same as $5 + (2 + 7) = 14$
$(3 \times 4) \times 2 = 24$ is the same as $3 \times (4 \times 2) = 24$

When working with different operations in the same problem, remember to perform the calculations in the BIDMAS order: Brackets, Indices, Division, Multiplication, Addition, Subtraction

Exercise 1.2

For each question write Equal to or Not equal to in order to make the sentence true.

1. $12 + 4 + 7$ is____________ $4 + 12 + 7$.

2. $5 \times 9 \times 8$ is____________ $8 \times 5 \times 9$.

3. $6 \times 10 \times 14 \times 9$ is____________ $10 \times 6 \times 6 \times 14$.

4. $25 + 17 + 8 + 34$ is____________ $17 + 34 + 8 + 25$.

5. $3 \times 7 + 13 \times 4 + 9$ is____________ $9 + 4 \times 13 + 3 \times 7$.

The distributive property

The distributive property refers to how numbers behave when an expression has multiplication together with addition or subtraction. The answer remains the same whether you do the addition or multiplication first. We use this particularly when simplifying expressions that include brackets.

Notice the two interchangeable operations in each example:
$2(4 + 7) = (2 \times 4) + (2 \times 7) = 8 + 14 = 22$ is the same as $2(4 + 7) = 2(11) = 22$.
$5(9 - 2) = (5 \times 9) - (5 \times 2) = 45 - 10 = 35$ is the same as $5(9 - 2) = 5(7) = 35$.

Method 1: Multiply the number outside the bracket by each number inside the brackets, then add/subtract the products.
Method 2: Add/subtract the numbers inside the brackets, then multiply the answer by the number outside the bracket.

Exercise 1.3

Simplify the expressions below. Do each calculation using both methods and check that you get the same answer.

1. $10(4 + 5)$
2. $12(5 - 2)$
3. $8(15 - 8)$
4. $11(7 + 11)$
5. $3(6 + 5)$
6. $9(63 - 52)$
7. $6(9 + 2)$
8. $3(50 - 25)$
9. $8(24 - 12)$

The multiplicative property of 1

The identity element for multiplication is 1. This means that any number multiplied by one equals the same number, so:
$5 \times 1 = 5$ $100 \times 1 = 100$ $x \times 1 = x$

The additive property of 0

The identity element for addition is 0. This means that any number plus zero equals the same number, or $x + 0 = x$

The divisive property of 0

When dividing by a number, you can use any number except for zero. In mathematics, you cannot divide a number by zero and we say that the answer is undefined.

Exercise 1.4

Calculate.

1. $12 \times 0 =$
2. $1 \times 42 =$
3. $25 - 0 =$
4. $0 \times 7 =$
5. $18 \times 0 =$
6. $0 - 3 =$
7. $16 \times 1 =$
8. $1 \times 10 =$
9. $1 - 0 =$
10. Use a calculator and type [1] [÷] [0] [=]. Write down what appears on the display. Explain why the calculator returns this answer.

Inverse operations

We can group the four basic operations into two pairs or inverse operations. Inverse operations are opposites and knowledge of them is useful when performing calculations. Inverse operations cancel out each other. When addition and subtraction cancel each other, the numbers are the same but the signs are opposite. The inverses of addition and subtraction give a result of zero:
$+1 - 1 = 0$ $+25 - 25 = 0$ $+x - x = 0$

When multiplication and division cancel out each other, the signs stay the same but the numbers flip from numerator to denominator of a fraction. The inverses of multiplication and division give a result of one:

$\frac{3}{1} \times \frac{1}{3} = \frac{3}{3} = 1$ $\frac{10}{1} \times \frac{1}{10} = \frac{10}{10} = 1$ $\frac{x}{1} \times \frac{1}{x} = \frac{x}{x} = 1$

> Remember that a fraction can also be written as a division: $\frac{2}{1} = 2 \div 1$ Any number can be written as a fraction with a denominator of 1.

Exercise 1.5

1. Use inverse operations to find the missing numbers

- **a)** ___ + 69 = 42
- **b)** 6 × ___ = 30
- **c)** ___ − 51 = 40
- **d)** 49 ÷ ___ = 7

2. Rewrite the following expressions as inverses and solve.
For example, 3 + 2 becomes 3 − 2 = 1

- **a)** 8 ÷ 9
- **b)** 75 + 46
- **c)** 12 ÷ 12
- **d)** 20 × 4
- **e)** (19 − 21) × 10
- **f)** 100(16 + 6)

Prime numbers

A prime number is any whole number that can be divided without remainder by only two numbers: 1 and itself. The first prime number is 2 because it can only be divided evenly by 1 and 2. The number 4 is not a prime number because it can be divided by 1, 2 and 4.

This is an exercise you can do to identify all the prime numbers from 1 to 100. You will need to use your knowledge of the multiplication tables up to 12 × 12.

> 1 is not a prime number because it can only be divided by one number, 1.

Exercise 1.6

Draw a table that is 10 blocks by 10 blocks. Write the numbers from 1 to 100 inside the blocks. Complete the following steps on your table:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

1. Cross out 1.
2. Cross out all the multiples of 2, but not 2 itself.
3. Cross out all the multiples of 3, but not 3 itself.
4. Cross out all the multiples of 4.
5. Cross out all the multiples of 5, but not 5 itself.
6. Cross out all the multiples of 6.
7. Cross out all the multiples of 7, but not 7 itself.
8. Cross out all the multiples of 8.
9. Cross out all the multiples of 9.
10. Cross out all the multiples of 10.
11. Cross out all the multiples of 11, but not 11 itself.
12. Cross out all the multiples of 12.

All the unshaded blocks are the prime numbers. List the prime numbers shown in your table.

Calculation techniques

Let's look at the different calculation techniques we can use when working with whole numbers.

Calculating in columns

For addition, subtraction and multiplication, we can set out our calculations in columns. We look at the place values of the numbers to be calculated and write one below the other, lining up the numbers of the same place value.

The required calculation is then performed on each column of place values and the subtotal is written below. When all the columns have been calculated, we add up the subtotals to find the final answer, which we write between two horizontal lines.

Examples

Work through each example of calculating in columns below. The calculations for each subtotal have been shown.

$$\begin{array}{r} 23 \\ +\ 108 \\ \hline 3+8=11 \\ 20+0=20 \\ 100 \\ \hline 131 \\ \hline \end{array} \qquad \begin{array}{r} 1\overset{1}{6}4 \\ -\ 72 \\ \hline 4-2=2 \\ +\ 160-70=90 \\ \hline 92 \\ \hline \end{array} \qquad \begin{array}{r} 18 \\ \times\ 34 \\ \hline 4\times 8=32 \\ 4\times 10=40 \\ 30\times 8=240 \\ +\ 30\times 10=300 \\ \hline 612 \\ \hline \end{array}$$

Long division

Long division is done in columns, but instead of adding up your subtotals, you subtract one subtotal from the next. This you do until there is no remainder.

Start by writing the dividend below a line and placing the divisor on its left. The calculations will be done below the dividend, and the quotient will be written along the top of the line. You will be working in place value columns, but starting from the left-hand side of the dividend.

The divisor is the number you are dividing by. The dividend is the number that you are dividing. The quotient is the answer to a division calculation.

Exercise 1.7

1. Calculate the following.
   a) $995 - 46$
   b) $261 + 479$
   c) $587 + 424 - 113$
   d) $86 + 587 + 321 - 694 + 25$
   e) 26×13
   f) 64×81
   g) $112 - 7$
   h) $225 \div 5 \times 5$
2. There are 62 learners in Class A, 47 learners in Class B and 58 learners in Class C.
   a) How many learners are there altogether?
   b) If each learner has two school books and five pencils, how many books and pencils do they have in total?

Rounding off and compensating

Rounding off and compensating is useful for adding and subtracting larger numbers. You first round off the number to make the calculation simpler, so you are adding to it. You then need to subtract the amount that you added, which is called compensating.

Examples

Calculate 875 + 596 by rounding off and compensating.
Round off 596 to the nearest 100 so it becomes 600.
875 + 600 = 1 475
1 475 – 4 = 1 471

Round off the number that is closest to 10, 100, 1 000 etc.

By rounding off we have added 4 to the amount.

Write the new sum and calculate a total.

Compensate by subtracting the amount that you added to round off, in this case 4.

Estimating

The focus in mathematics is often about getting to an exact answer, but in practice we sometimes need to be able to make calculations in a hurry. In these situations, we are looking for an answer that is not exact but one that is good enough and can be worked out quickly. We consider the numbers in the calculation and round off one or both of them to make the calculation easier. Because we are not looking for an exact answer in estimating, we don't compensate for the rounding off, but rather we try to keep it as minimal as possible.

Examples

A man is driving a journey of 854 km from point A to point B. If one litre of petrol will give him 12 km, estimate how many litres of petrol he will need.
854 km ÷ 12 km/l
850 km ÷ 10 km/l = 85 litres

Write a calculation to describe the problem.

Round off both numbers to the nearest 10 and calculate the answer in litres.

Calculators

As you progress in the Senior Phase in Mathematics, you will have to rely more and more on using a calculator. It is important that you know how to operate your calculator properly because it can save you a lot of time when you do calculations. Calculators are also useful for checking calculations that you have done manually.

Check the instruction booklet that came with your calculator, or use the Internet to find an online booklet.

Exercise 1.8

Use a calculator to determine the following.

1.

- **a)** 884 756 + 56 896 +207 189
- **b)** 501 487 – 99 509 – 274 642
- **c)** 23 748 + 96 470 – 84 192
- **d)** 28 × 71
- **e)** 1 560 – 39
- **f)** 96 × 54
- **g)** 5 063 – 61
- **h)** 34(65 822 – 64 912) – 28 648
- **i)** (82(784 – 28) + 4) – 50
- **j)** (754 390 – 692 482 – 61 283) – 25
- **k)** 74 × 26 + 96 – 32 + 49 × 32
- **l)** (38 × 507 – 65 × 216) – 871

2. Estimate the answers to the following as quickly as you can. Then, use a calculator and write down the accurate answer next to your estimate.

- **a)** 426 783 + 586 021 – 678 932
- **b)** 696 × 33
- **c)** 854 980 – 124
- **d)** 11 × 784
- **e)** 16 426 – 382
- **f)** (903 462 – 766 414) – 74 × 378